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LETTER TO THE EDITOR

Bohr–van Leuwan theorem for non-magnetization and plasma non-confinement

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Abstract. The Bohr–van Leuwan theorem, relevant to solid state physics, which concerns magnetization in the classical limit, is reviewed. This derivation, which is based on statistical mechanical evaluation of the free energy, is generalized and applied to the case of a confined plasma. It is shown that an equilibrium, electromagnetically confined classical plasma at a given temperature and occupying finite volume does not exist. The derivation is valid for external as well as self-field confinement.

We consider an electromagnetically confined classical plasma in equilibrium at a given temperature T which occupies a finite volume, V. If the system obeys classical Hamiltonian dynamics, then it is concluded that the said equilibrium configuration does not exist. This statement is shown to be valid for both external or self-confinement. In this context the present result is a generalization of that obtained by Schmidt [1] in which a plasma virial form [2] was employed to demonstrate that plasma self-confinement does not exist. This result is often quoted in plasma works [3–6].

Schmidt's analysis [1] is based on the macroscopic fluid dynamical 'momentum equation' [7]. In contrast, the present analysis stems from the free-energy partition function of a plasma in an electromagnetic field which involves the Hamiltonian of the system. This formulation of the problem readily permits an external confinement force to be included in the formalism.

The present analysis is an extension of the Bohr–van Leuwan theorem [8, 9] which states that the magnetization of a classical system vanishes. It is important to the present study to offer a brief review of this derivation.

From statistical mechanics [10, 11] the free energy, F, of a classical system is given by

$$\exp(-\beta F)=\int\dots\int \mathrm{d}1\,\mathrm{d}2\dots\mathrm{d}N\exp[-\beta H(1,2,\dots,N)] \tag{1}$$

where $\beta\equiv 1/k_{\mathrm{B}}T$ and '1' $\equiv(\boldsymbol{x}_1,\boldsymbol{p}_1)$ represents the phase coordinates of particle number 1, and H is the Hamiltonian of the system. The magnetic field is present in H through terms of the form $\boldsymbol{p}_i+(e/c)\boldsymbol{A}(\boldsymbol{x}_i)$ where $\boldsymbol{A}(\boldsymbol{x}_i)$ is the vector potential at the position of the ith particle in the system. Introducing the transformation,

$$\boldsymbol{p}_i+\frac{e}{c}\boldsymbol{A}(\boldsymbol{x}_i)=\boldsymbol{p}_i' \tag{2}$$

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and integrating over transformed momenta leaves the resulting coordinate integral of (1) independent of magnetic field (with components of momenta in the interval $(-\infty, +\infty)$). Thus, the magnetization

$$M = -\frac{\partial F}{\partial H} = 0 \tag{3}$$

where H is magnetic field. The preceding represents the conclusion of the Bohr–van Leuwan theorem.

Now consider a plasma which is in equilibrium at temperature T and confined to a finite region of space of volume V, by an electromagnetic field. The Hamiltonian of the system is given by

$$H = \sum_i \frac{1}{2m_i}\left(p_i + \frac{e_i}{c} A(x_i)\right)^2 + \sum_i \Psi(x_i) \tag{4}$$

$$\Psi(x_i) = \Phi(x_i) + \frac{1}{2}\sum_{j\neq i} \varphi_{i,j}(x_i, x_j) \tag{4a}$$

where $\varphi_{i,j}$ represents interparticle interaction. The vector potential in (4) includes both self and externally supported magnetic fields whereas the term $\Phi(x_i)$ represents any additional externally supported fields.

When (4) is inserted in the integral of (1), the vector potential terms are eliminated through the transformation (2), whereas the remaining potentials are purely coordinate dependent and are integrated out of the expression. Here it is assumed that constants stemming from external fields or cut-offs in the interparticle potential appear as multiplicative factors in the expression for the partition function. It follows that the free energy corresponding to the plasma Hamiltonian (4) may then be written

$$F = F(T, V) + \text{constant}. \tag{5}$$

The pressure of the plasma,

$$P = -\left(\frac{\partial F}{\partial V}\right)_T \tag{6}$$

is therefore independent of the fields. By standard thermodynamic interpretation [12], the pressure given by (5) is that exerted on the medium by the surroundings. In the present case, this is the pressure of the plasma on the confining fields which, with our preceding finding, is independent of the fields. This conclusion invalidates our starting assumption of an electromagnetically confined finite equilibrium plasma. We may conclude that no such equilibrium configuration exists. As is evident from (4), (4a) this conclusion is valid for both external or self-field confinement and in this context, as noted above, is a generalization of a previous result based on the plasma virial.

In conclusion we have found that the Bohr–van Leuwan theorem finds application to both classical magnetization and electromagnetic confinement of a classical thermodynamic plasma. Our demonstration of the non-existence of an equilibrium confined finite plasma was shown to be valid for both external and self-field confinements.

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